

# 14-1 HINGE RESTRAINER DESIGN METHOD

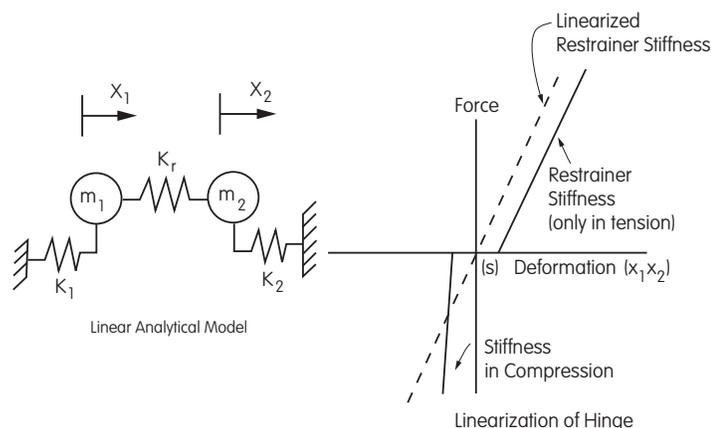
## Introduction

Bridge Design Aid 14-1 provides a simplified method for designing cable restrainers for bridges. It is based on a research study using a simplified two-degree-of-freedom linear analytical model. Twenty six different earthquake records were utilized and verified through non-linear analysis using ground motion records scaled to  $0.7g$ <sup>1</sup>. Some of the advantages of this method are:

- Uses a simplified two-degree-of-freedom linear analytical model
- Single-step process - no iterations required
- No modal analysis required
- Conservative for most known earthquake ground motions
- Simplified ground motion period ( $T_g$ ) determination compatible with the Seismic Design Criteria (SDC) soil types

## Assumptions:

- The effects of friction are negligible
- The skew is  $\leq 30$  degrees. For bridges with skews  $> 30$  degrees, the lead office should consult with the Office of Earthquake Engineering (OEE) for guidance.
- The hinge is represented by a linearized model as shown in Figure 1:



**Figure 1**

<sup>1</sup> UC Berkeley Report No. UCB/EERC 97/12. "New Design and New Analysis Procedures for Intermediate Hinges in Multiple - Frame Bridges."

## Definitions of Variables

- $A$  = Restrainer cross-sectional area
- $c$  = The effective modal damping factor
- $c_{avg}$  = The average effective modal damping factor of frames 1 and 2
- $c_i$  = The effective modal damping factor for frame  $i$
- $D_1$  = The displacement demand of the less flexible frame
- $D_2$  = The displacement demand of the more flexible frame
- $D_{eq0}$  = The unrestrained relative hinge displacement
- $D_i$  = The spectral displacement demand of frame  $i$
- $D_{ia}$  = The spectral displacement of frame  $i$  adjusted by a damping reduction factor  $R_{di}$
- $D_r$  = The restrained relative hinge displacement
- $D_y$  = The yield (max) elongation the restrainer is expected to experience in a seismic event
- $E$  = Restrainer modulus of elasticity
- $f$  = An effective stiffness adjustment factor
- $F$  = Adjusted effective stiffness factor
- $F_{eff}$  = Effective stiffness factor
- $F_{yi}$  = Yield force of frame  $i$
- $G$  = Soil shear modulus
- $G_{max}$  = Maximum soil shear modulus
- $H$  = Soil depth
- $K_1$  = The stiffness of the less flexible frame
- $K_2$  = The stiffness of the more flexible frame
- $K_{1eff}$  = The effective stiffness of the less flexible frame
- $K_{2eff}$  = The effective stiffness of the more flexible frame
- $K_{ieff}$  = The effective stiffness of the frame  $i$
- $K_i$  = The stiffness of the frame  $i$
- $K_{mod}$  = The reciprocal of the sum of flexibilities of frames 1 and 2
- $K_r$  = The restrainer stiffness

- $K_{min}$  = The minimum restrainer stiffness required if unseating is possible during the elastic stage
- $l$  = Length of restrainer
- $L$  = Relative hinge displacement limit
- $m_i$  = Mass of frame “i”
- $N$  = Number of restrainers
- $r$  = An adjustment factor for  $R$
- $R$  = Restraint level factor
- $R_d$  = Displacement reduction factor ( SDC 2.1.5 )
- $R_{di}$  = Displacement reduction factor for frame i ( SDC 2.1.5 )
- $s$  = Restrainer slack
- $T_1$  = The fundamental (natural) period of the less flexible frame
- $T_2$  = The fundamental (natural) period of the more flexible frame
- $T_g$  = The ground period
- $T_{1eff}$  = The effective fundamental (natural) period of the less flexible frame
- $T_{2eff}$  = The effective fundamental (natural) period of the more flexible frame
- $T_{ieff}$  = The effective fundamental (natural) period of frame i
- $T_i$  = The fundamental (natural) period of frame i
- $V_s$  = The shear wave velocity
- $\beta$  = The effective period ratio  $T_{2eff} / T_{1eff}$
- $\Delta y_i$  = Yield displacement of frame i
- $\mu_d$  = The expected displacement ductility demand, default = 4.0
- $\mu_{avg}$  = Average displacement ductility demand of frames 1 and 2
- $\rho_{12}$  = Modal cross-correlation coefficient

### Design Theory:

The restrainer stiffness,  $K_r$  required for controlling the unrestrained relative hinge displacement  $D_{eq0}$ , may be determined by:

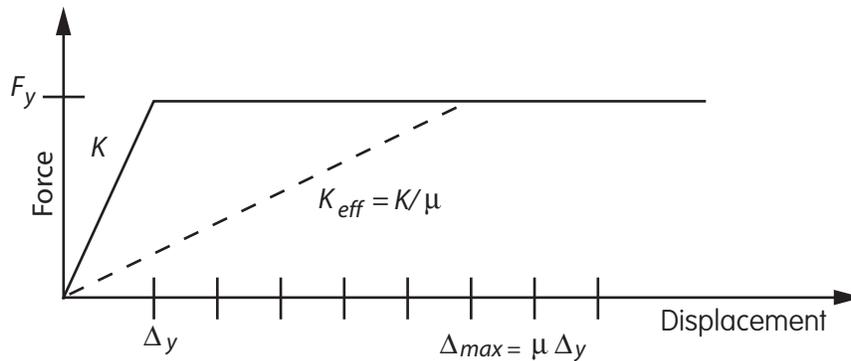
$$K_r = R F K_{mod.} \quad \text{Eq. (1.1)}$$

The value of the restrained relative hinge displacement,  $D_r$  should be selected based on the purpose of restrainers such as:

- Protect the bearings and seals of the bridge in a moderate event,
- Prevent unseating from a short seat where seat extensions are not practical,
- Control the response of the structure in a major event.

This method utilizes the effective stiffness concept to determine the effective period and spectral displacements of the yielding frames:

For  $K_{eff}$  (effective stiffness), see Figure 2



**Figure 2**

## Hinge Restrainer Design Procedure

### Determine $N$

1) Determine  $D_{eq0}$  :

For all frames “ $i$ ” determine

$m_i$  (slugs)

$K_i$  (kips/in)

$$T_i = 2\pi\sqrt{(m_i / K_i)} \quad (\text{sec}) \quad \text{Eq. (1.2)}$$

$\Delta y_i$  (in)

$\mu_d$

$F_{yi}$  (kips)

For all frames “ $i$ ” calculate effective values:

$$K_{ieff} = K_i / \mu_d \quad \text{Eq. (1.3)}$$

$$T_{ieff} = 2\pi\sqrt{(m_i / K_{ieff})} \quad \text{Eq. (1.4)}$$

Note: if  $\mu_d$  is not known use a default value of 4.0

For all frames “ $i$ ” determine  $D_i$  (from ARS curves<sup>2</sup>),  $c_i$ ,  $R_{di}$ , and  $D_{ia}$  :

$D_i$  = Spectral Displacement based on ( $T_{ieff}$ )

$$c = 0.05 + [1 - (0.95/\sqrt{\mu}) - 0.05\sqrt{\mu}] / \pi \quad (\text{See Figure 3}) \quad \text{Eq. (1.5)}$$

$$R_{di} = [1.5 / (40c + 1)] + 0.5 \quad (\text{See Figure 3}) \quad \text{Eq. (1.6)}$$

$$D_{ia} = R_{di} D_i \quad \text{Eq. (1.7)}$$

For adjacent frames 1 and 2 determine:

$$\beta = T_{2eff} / T_{1eff} \quad \text{Eq. (1.8)}$$

$$\rho_{12} = \frac{8c_{avg}^2 (1 + \beta)(\beta)^{3/2}}{(1 - \beta^2)^2 + 4c_{avg}^2 \beta(1 + \beta)^2} \quad (\text{See Figure 4}) \quad \text{Eq. (1.9)}$$

---

<sup>2</sup> ARS curves are for 5% damping. Spectral displacements should be adjusted for other values.

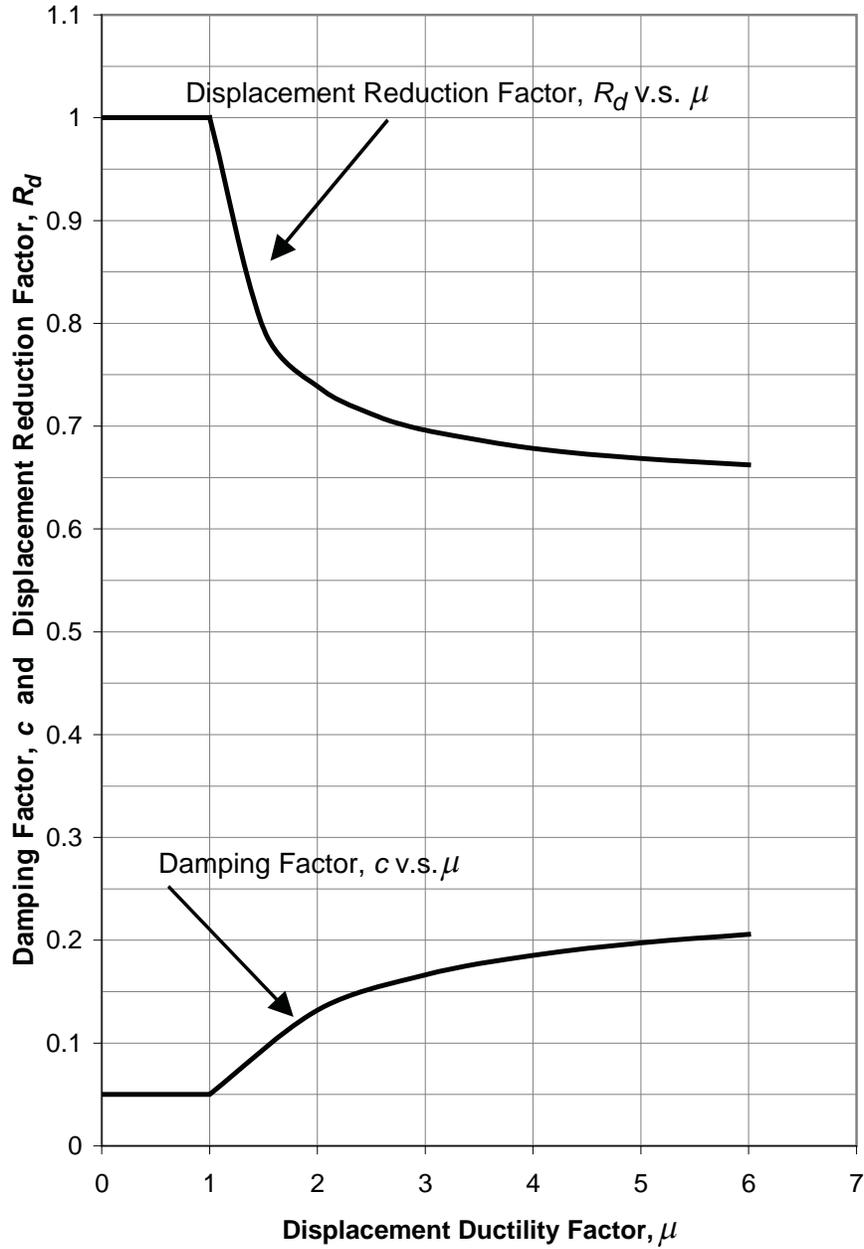


Figure 3

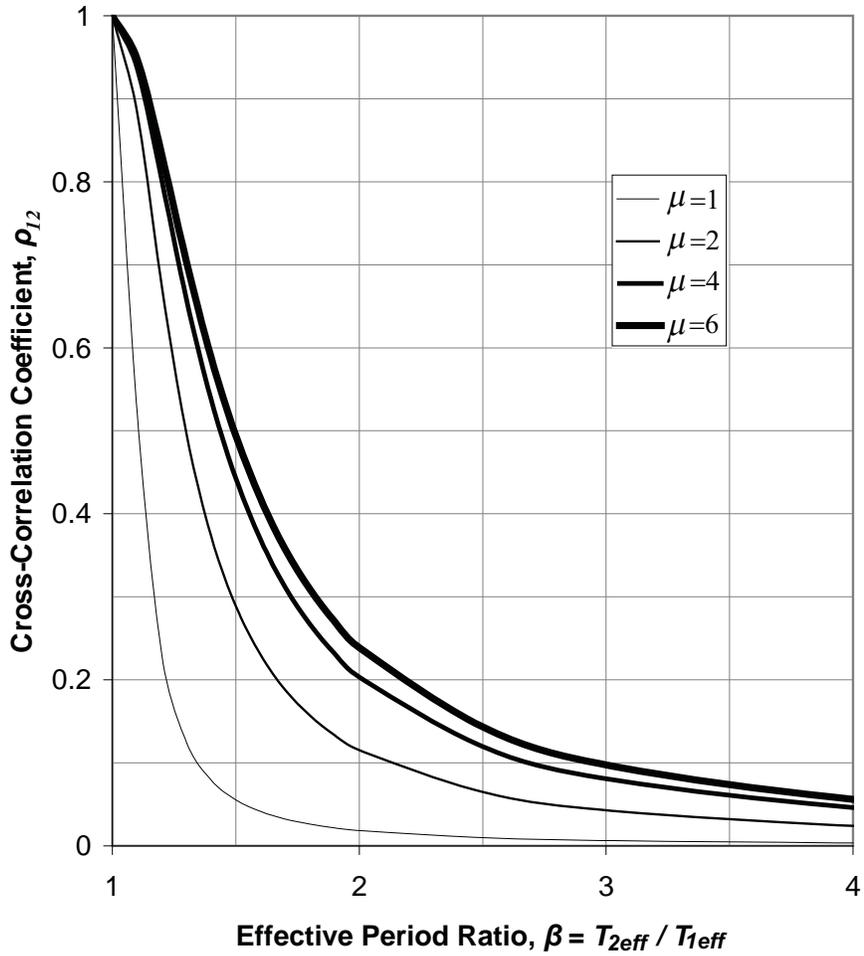


Figure 4

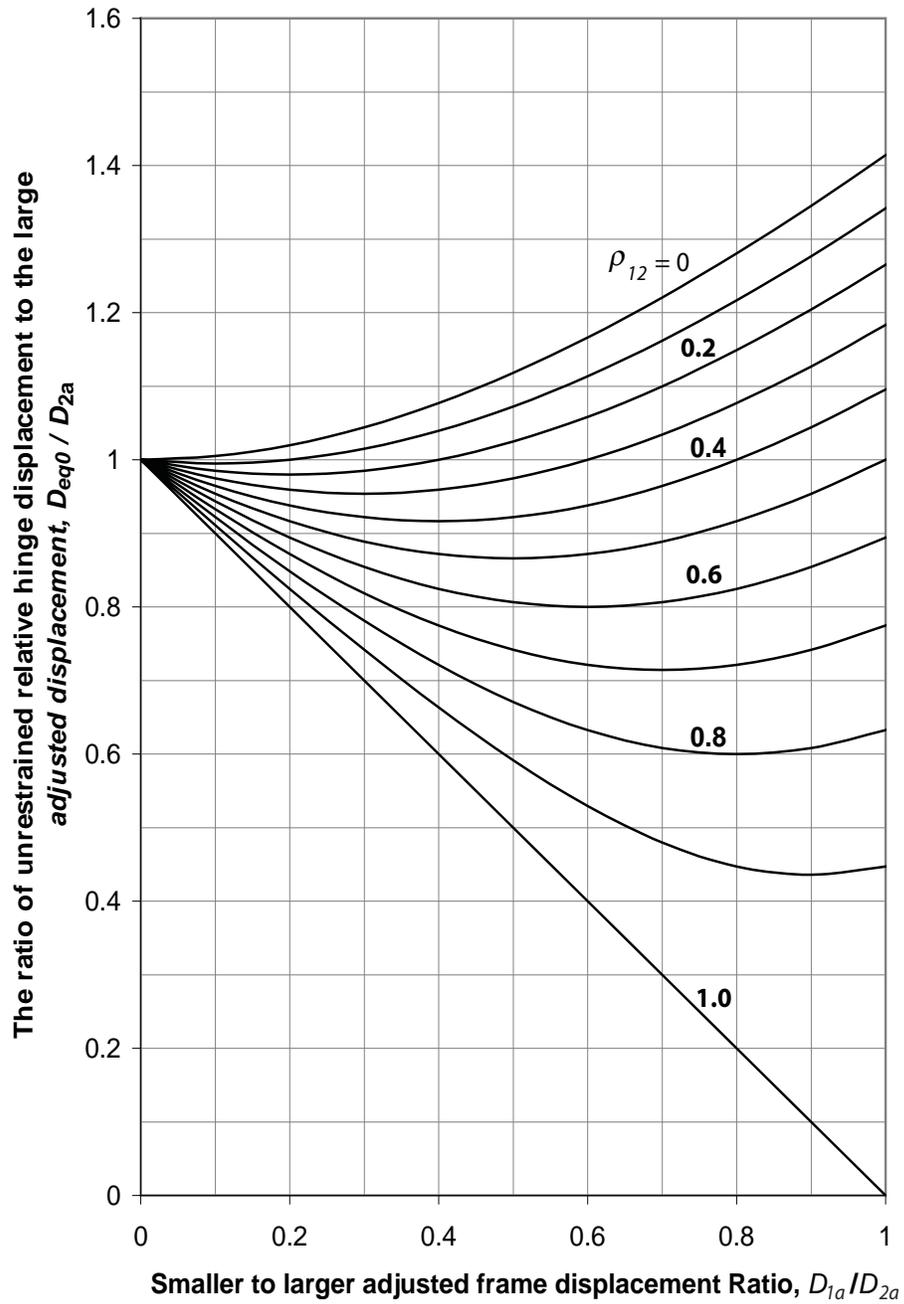


Figure 5

Calculate  $D_{eq0}$  :

$$D_{eq0} = \sqrt{(D_{1a}^2 + D_{2a}^2 - 2\rho_{12}D_{1a}D_{2a})} \quad (\text{See Figure 5}) \quad \text{Eq. (1.10)}$$

2) Determine  $R$

Find the allowable or desired restrained relative hinge displacement,  $D_r$

$$D_r = D_y + s \quad \text{Eq. (1.11)}$$

Calculate the displacement limit,  $L$

$$L = D_r / D_{eq0} \quad \text{Eq. (1.12)}$$

Calculate  $r$ ,

$$r = -L + 1.5 \quad \text{Eq. (1.13)}$$

Calculate  $R$ ,

$$R = r ( 1 - 1.66L + 0.67/L ) \quad (\text{See Figure 6}) \quad \text{Eq. (1.14)}$$

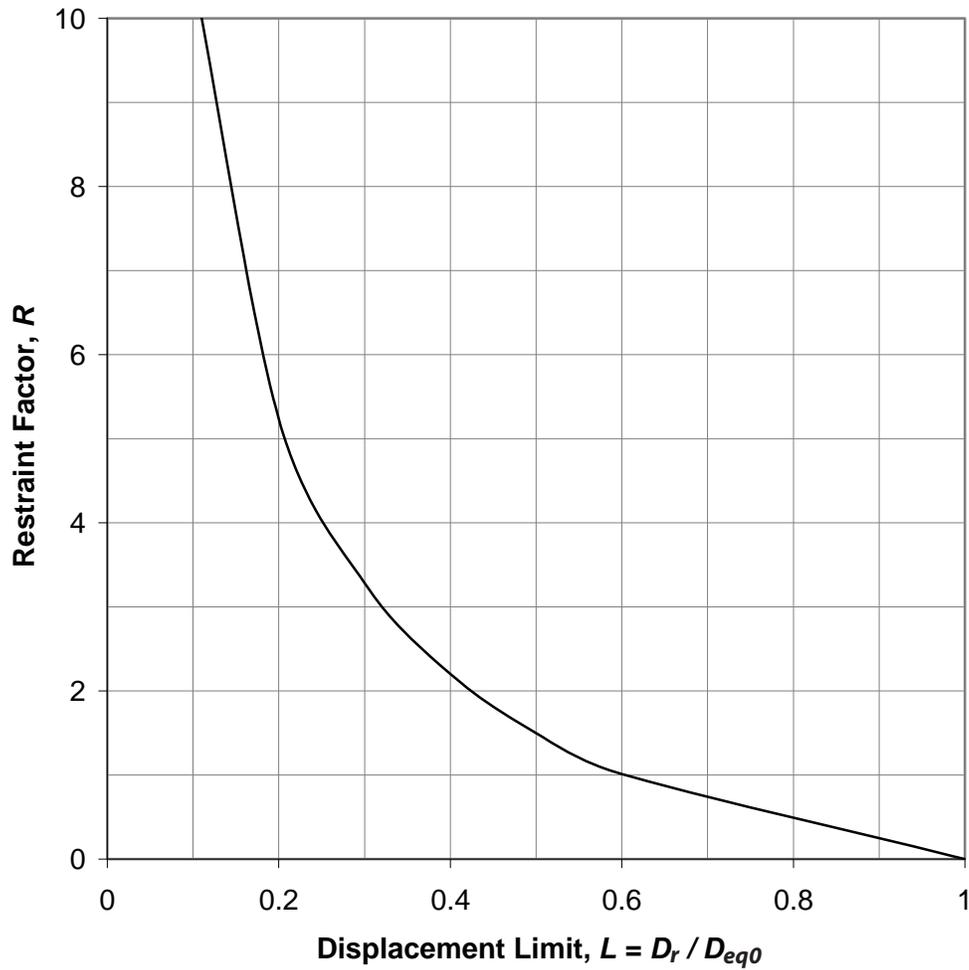


Figure 6

3) Determine  $F$

Determine  $T_g$ ,

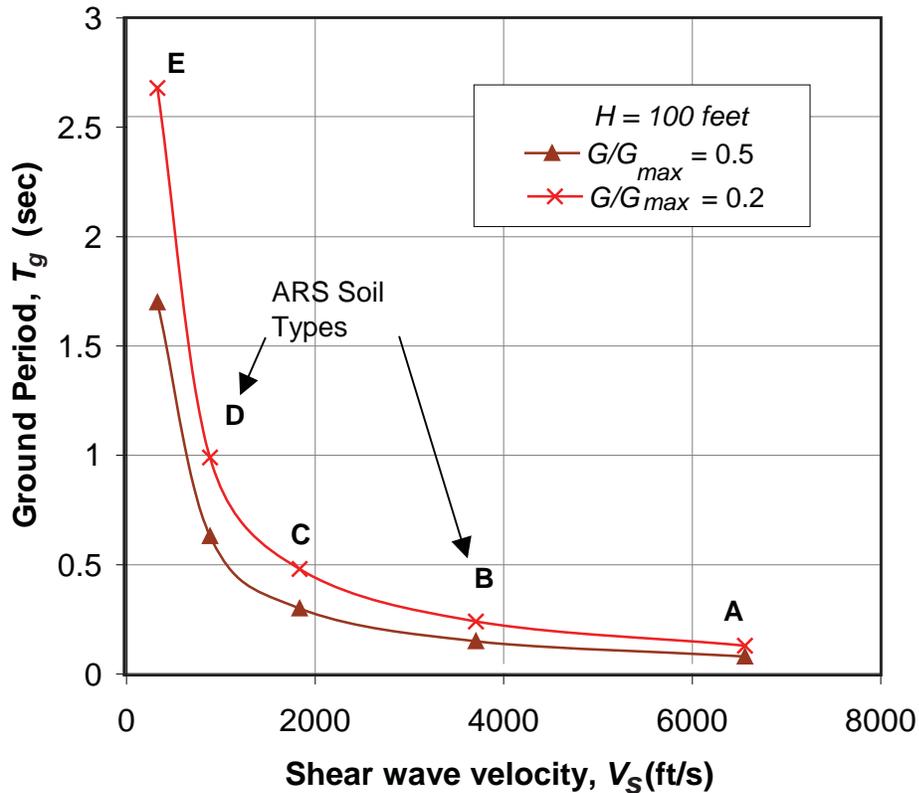
$$T_g = 4H / \left[ V_s \sqrt{(G/G_{max})} \right] \quad (\text{See Figure 7}) \quad \text{Eq. (1.15)}$$

Calculate  $T_2/T_g$

Determine  $F_{eff}$  (See Figure 8)

Determine the adjustment factor,  $f$  (See Figure 9)

Calculate  $F = f \times F_{eff}$



Note:  $G/G_{max}$  varies inversely to ground motion intensity

Figure 7

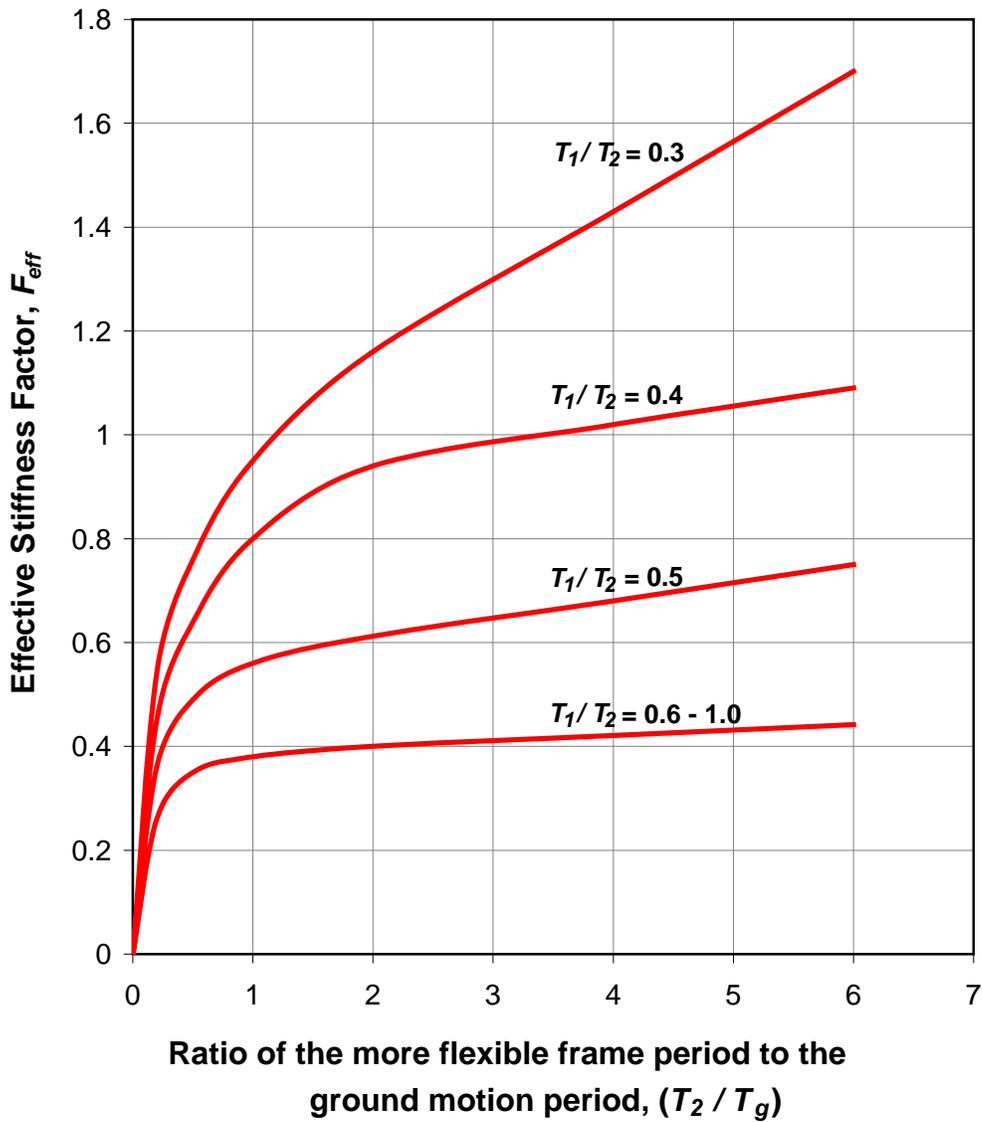


Figure 8

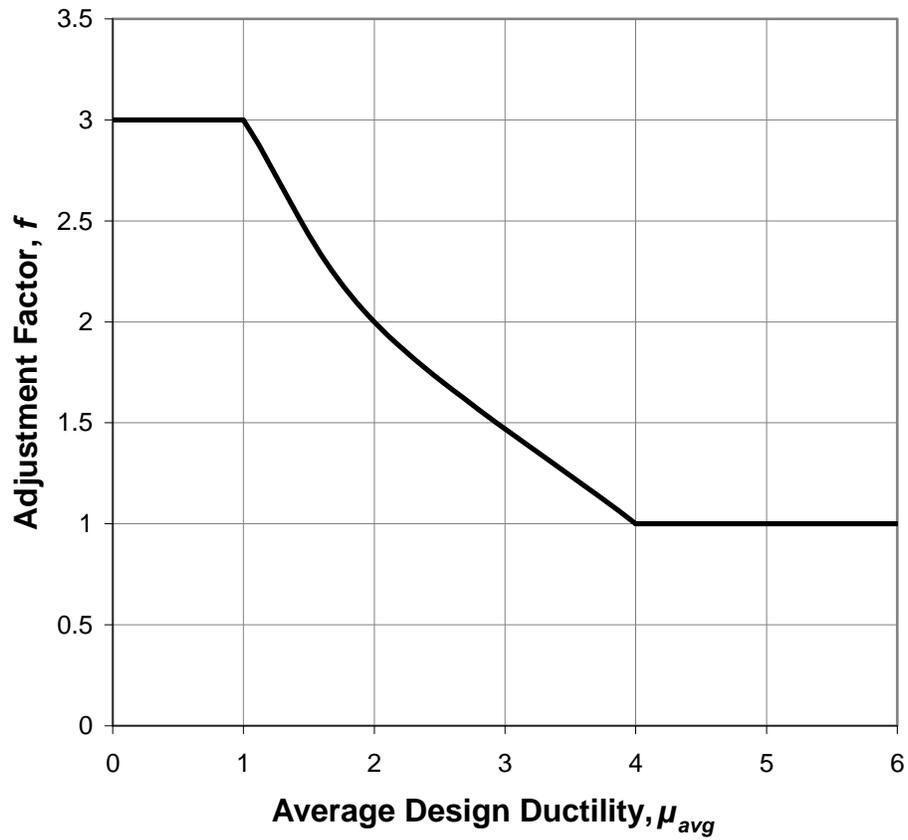


Figure 9

4) Determine  $K_{mod}$

Calculate  $K_{mod} = K_1 K_2 / (K_1 + K_2)$  (See Figure 10)

Eq. (1.16)

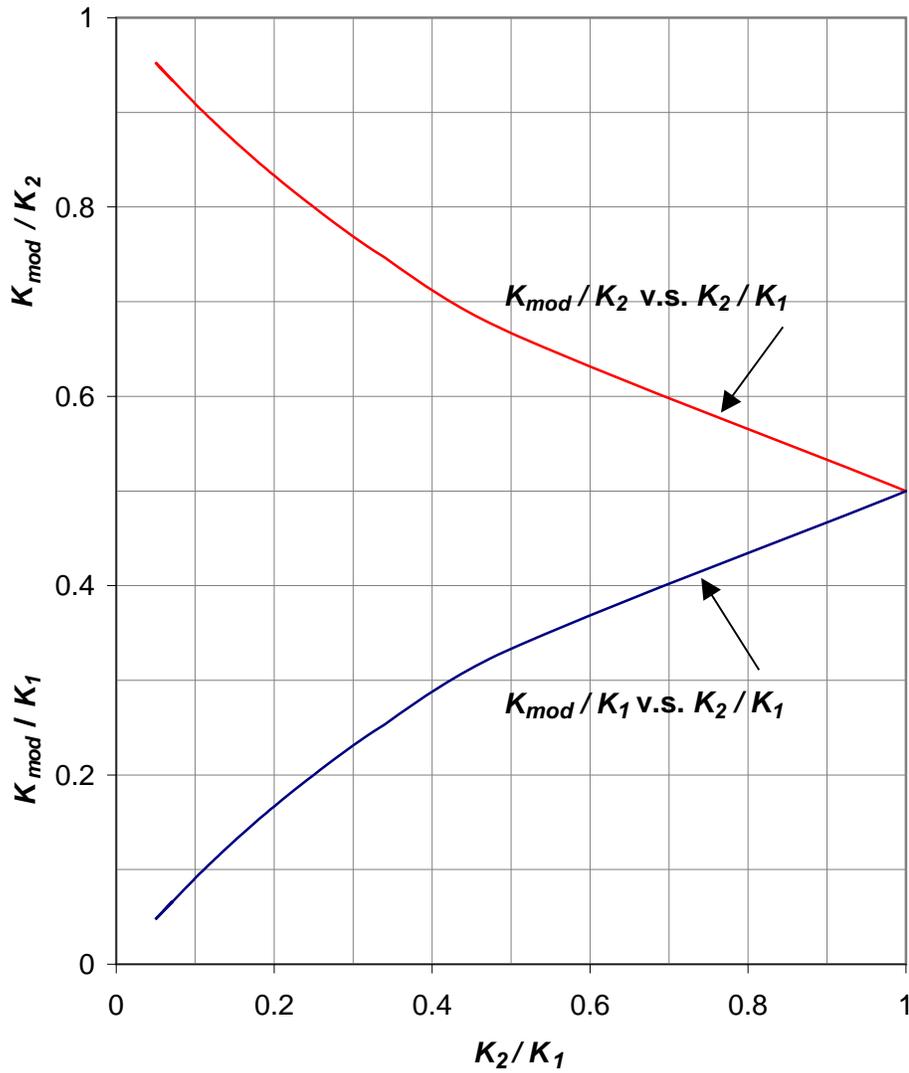


Figure 10

- 5) Calculate the required restrainer stiffness  $K_r$  using Eq. (1.1)
- 6) Check if  $K_{min}$  requirement applies

If  $D_{eq0} - D_y > \Delta_{y2}$  then continue, otherwise, proceed to Step 7 Eq. (1.17)

Calculate  $K_{rmin} = F_{y2} / D_y$

If  $K_{rmin} > K_r$  calculated in step 5 above then  $K_{rmin}$  controls and

use  $K_r = K_{rmin}$

If  $K_r > K_{rmin}$  then use  $K_r$

7) Calculate the number of restrainers,  $N$

$N = K_r l / EA$  Eq. (1.18)

## Restrainer Design for Multi-Span Simply Supported Bridges

The simple supports of multi-span simply supported bridges (SSB) can be modeled similar to in-span hinges of multi-frame structures. The required restrainer stiffness may be determined using the same method.

To model a simple support of a SSB, each joint should be considered individually (Note that every internal support contains two joints, one for each simple span it supports). Assume that the selected joint is the only joint that allows movement (hinge) while all other joints act as pins. This results in two frames, one to the left, and the other to the right of the selected joint (hinge). This will include several scenarios, with frames defined on either side of the hinge. The number of spans and columns included in each frame is determined by the joint modeling procedure shown in the next section.

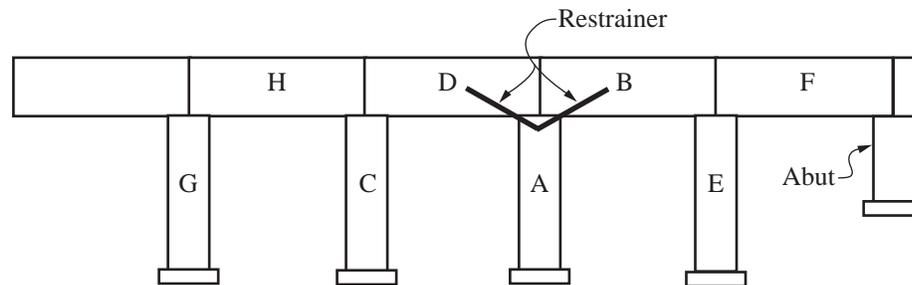
For typical bridges assume it is not necessary to consider more than two spans and two columns on either side of the hinge for any particular scenario (see Figure 11). This results in a maximum of four scenarios for every hinge. For each scenario, the stiffnesses, periods, displacement demands, hinge opening and the corresponding required restrainer stiffness are determined in the same manner as for in-span hinges. The scenario that results in the largest hinge opening will control. If the hinge opening demand is greater than or equal to the available seat length, restrainers must be provided as required or may be added for performance<sup>3</sup>. The same procedure is repeated for the second joint of the same support and the required restrainer stiffness is calculated. Apply the same procedure to all other supports. Abutments should be considered fixed.

---

<sup>3</sup>. Memo to Designers 20-4 requires retrofit for all seat lengths  $\leq 12$  inches.

Joint Modeling Procedure Illustration:

The structure in Figure 11 illustrates the joint modeling procedure:



**Figure 11**

- 1.) For retrofits, if the support is a pin then the strength of the bearing must be evaluated and if the strength is greater than the earthquake force then no restrainers are required to prevent unseating. Restrainers may be added if performance or bearing protection is required. If the support bearing strength is less than the seismic force then it should be treated as a roller.
- 2.) To design restrainer AD consider the two frames:

Scenario 1:

Frame AB (column A and span B) moving right

Frame CD (column C and span D) moving left

Calculate the periods of both frames AB and CD and the required restrainer stiffness utilizing the same method for intermediate hinges of multi-span bridges.

Similarly check other scenarios for tension and compression models (no need to check more than two columns and spans on each side of the joint)

Scenario 2:

Frame ABEF moving right

Frame CD moving left

Scenario 3:

Frame AB moving right

Frame CDGH moving left



Scenario 4:

Frame ABEF moving right

Frame CDGH moving left

The situation that results in the most restrainers controls the design at that particular joint (hinge)

- 3.) To design restrainer AB investigate the two frames:

Frame AD (column A and span D) moving left

Frame BE (column E and span B) moving right

Calculate the periods of both frames AD & BE, the required restrainer stiffness and check other tension and compression models similar to restrainer AD.

- 4.) Repeat steps 2 and 3 for all intermediate supports.

- 5.) For Abutment supports (end spans):

Always model the abutment as fixed and  $T_1/T_2$  may be assumed to be 0.3 for simplicity since it represents a relatively very rigid frame (abutment like) adjacent to a flexible frame and the charts will be applicable without further analysis.

Scenario 1:

Frame 1 = Abutment (fixed)

Frame 2 = Frame EF (moving left)

Scenario 2:

Frame 1 = Abutment (fixed)

Frame 2 = Frame EFAB (moving left)

- 6.) For the restrainer EF, the frame consisting of span F and the abutment shall be considered fixed and treated as the Frame 1 above.

Scenario 1:

Frame 1 = Abutment + span F (fixed)

Frame 2 = Frame EB (moving left)

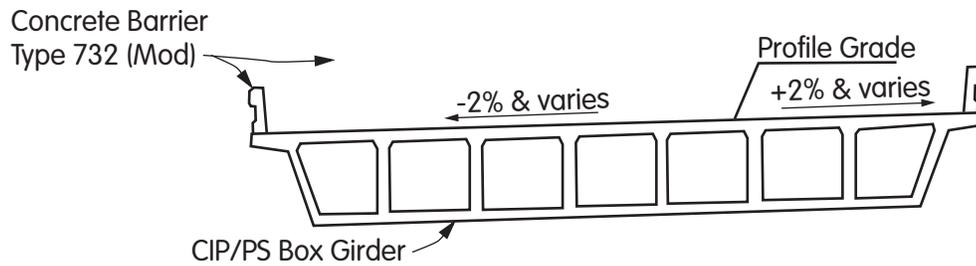
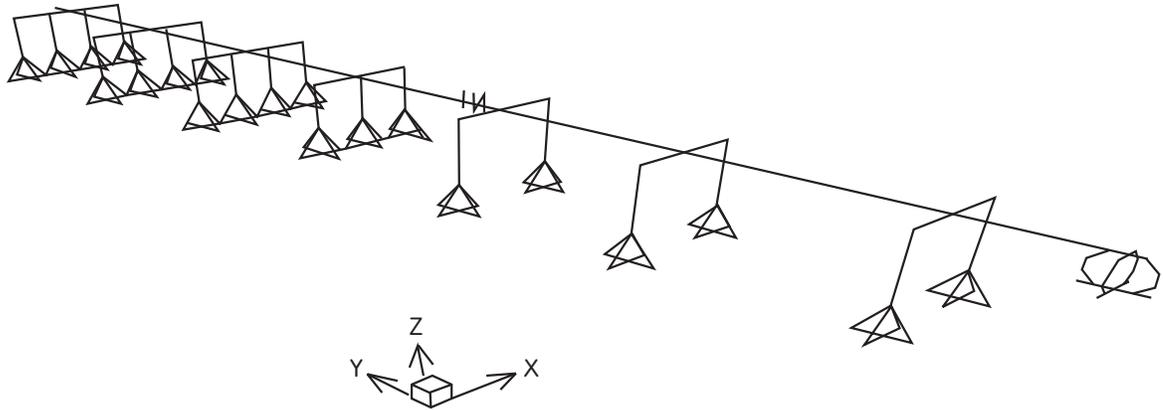
Scenario 2:

Frame 1 = Abutment + span F (fixed)

Frame 2 = Frame EBAD (moving left)

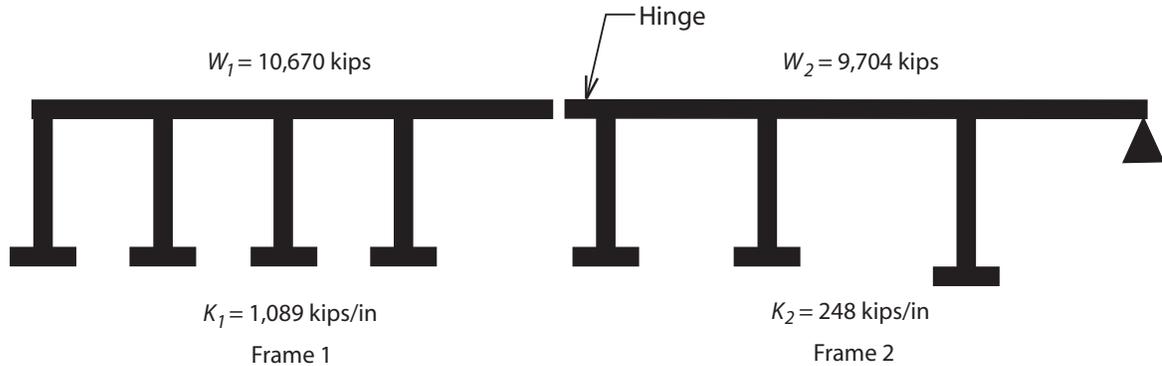
## Example Calculation for In-Span Hinges:

Determine the number of restrainers required for the following structure:



**Given:**

The structure is in a 0.7g seismic zone with a type C soil as described in the Caltrans Seismic Design Criteria (SDC). The structure has 6 inch hinge seats with a 4 inch allowable movement. The following sketch summarizes additional given information for each frame:



- 1) Determine  $D_{eq}$ :

For all frame, determine:  $m_i$ ,  $K_i$ ,  $T_i$ ,  $D_i$  (ARS curves),  $\Delta y_i$ ,  $\mu_{di}$ , and  $F_{yi}$

$$m_1 = 27.61 \text{ slugs}$$

$$m_2 = 25.11 \text{ slugs}$$

$$K_1 = 1089 \text{ kips/in}$$

$$K_2 = 248 \text{ kips/in}$$

$$T_1 = 2\pi\sqrt{27.61/1089} = 1 \text{ sec}$$

$$T_2 = 2\pi\sqrt{25.11/248} = 2 \text{ sec}$$

$$\Delta y_1 = 3.36 \text{ inches (given)}$$

$$\Delta y_2 = 4.17 \text{ inches (given)}$$

$$\mu_{d1} = 3.59 \text{ (given)}$$

$$\mu_{d2} = 5.34 \text{ (given)}$$

$$F_{y1} = 1089(3.36) = 3,659 \text{ kips}$$

$$F_{y2} = 248(4.17) = 1,034 \text{ kips}$$

Apply Eq. (1.3) calculate  $K_{ieff}$

Apply Eq. (1.4) calculate  $T_{ieff}$

$$K_{1eff} = 1,089/3.59 = 303.3 \text{ kips/in}$$

$$T_{1eff} = 2\pi\sqrt{(27.61/303.3)} = 1.9 \text{ sec}$$

$$K_{2eff} = 248/5.34 = 46.4 \text{ kips/in}$$

$$T_{2eff} = 2\pi\sqrt{(25.11/46.4)} = 4.6 \text{ sec}$$

For all frame "i" determine  $D_i$  (ARS curves),  $c_i$ ,  $R_{di}$ , and  $D_{ia}$  :

$D_1$  = Spectral Displacement ( at 1.9 sec ) = 21 in

$D_2$  = Spectral Displacement ( at 4.6 sec ) = 32 in

Apply Eq. (1.5) in Figure 3:

$$c_1 = 0.05 + \left[ 1 - \left( 0.95 / \sqrt{3.59} \right) - 0.05 \sqrt{3.59} \right] / \pi \cong 0.18$$

$$c_2 = 0.05 + \left[ 1 - \left( 0.95 / \sqrt{5.34} \right) - 0.05 \sqrt{5.34} \right] / \pi = 0.20$$

Apply Eq. (1.6) in Figure 3:

$$R_{d1} = [ 1.5 / ( 40(0.18) + 1 ) ] + 0.5 = 0.68$$

$$R_{d2} = [ 1.5 / ( 40(0.20) + 1 ) ] + 0.5 = 0.67$$

Apply Eq. (1.7):

$$D_{1a} = 0.68 (21) = 14.3 \text{ in}$$

$$D_{2a} = 0.67 (32) = 21.4 \text{ in}$$

For adjacent frames 1 and 2 determine  $\beta$  and  $\rho_{12}$

Apply Eq. (1.8):

$$\beta = 4.6 / 1.9 = 2.42$$

Calculate  $c_{avg}$  :

$$c_{avg} = (0.18 + 0.20) / 2 = 0.19$$

Apply Eq. (1.9) in Figure 4:

$$\rho_{12} = \frac{8(0.19)^2 (1+2.42)(2.42)^{3/2}}{(1-2.42^2)^2 + 4(0.19)^2 (2.42)(1+2.42)^2} = 0.134$$

Calculate the unrestrained relative hinge displacement,  $D_{eq0}$

Apply Eq. (1.10), Figure 5:

$$D_{eq0} = \sqrt{(14.3^2) + (21.4^2) - 2(0.134)(14.3)(21.4)} = 24.1 \text{ in}$$

2) Determine  $R$ :

Calculate  $D_r$ ,  $L$ ,  $r$ , and  $R$

Apply Eq. (1.11):

$$D_r = 3 + 1 = 4 \text{ in}$$

Apply Eq. (1.12):

$$L = 4 / 24.1 = 0.166$$

Apply Eq. (1.13), Figure 6:

$$r = -0.166 + 1.5 = 1.33$$

Apply Eq. (1.14), Figure 6:

$$R = 1.33 (1 - 1.66(0.166)) + 0.67/0.166 = 6.33$$

3) Determine  $F$ :

Determine  $T_g$ ,  $F_{eff}$ ,  $f$ , and  $F$ :

From Figure 7 (assume high ground motion intensity  $G/G_{max} = 0.2$ ):

$$T_g = 0.5 \text{ sec}$$

Calculate  $T_2 / T_g$  and  $T_1 / T_2$ :

$$T_2 / T_g = 2 / 0.5 = 4$$

$$T_1 / T_2 = 1 / 2 = 0.5$$

Determine  $F_{eff}$ :

From Figure 8,  $F_{eff} = 0.68$

Determine  $f$ :

Average  $\mu_d = (3.59 + 5.34) / 2 = 4.47 > 4.0$

From Figure 9,  $f = 1.0$

Calculate  $F$ :

$F = f \times F_{eff} = 1.0 \times (0.68) = 0.68$

4) Determine  $K_{mod}$

Apply Eq. (1.16), Figure 10:

$K_{mod} = 1089 (248) / (1089 + 248) = 202$  kips/in

5) Determine the required restrainer stiffness,  $K_r$

Apply Eq. (1.1):

$K_r = 6.33 (0.68)(202) = 870$  kips/in

6) Check if  $K_{rmin}$  requirement applies

Apply Eq. (1.17):

$D_{eq} - D_y = 24.1 - 3 = 21.1 > \Delta y_2 = 4.17$

$\therefore K_{rmin}$  applies

Calculate  $K_{rmin} = F_{y2} / D_y$

If  $K_{rmin} > K_r$  calculated in step 5 above then  $K_{rmin}$  controls

$K_{rmin} = 1034 / 3 = 345$  kips/in  $< 870$  ok use  $K_r$



7) Determine the required number of restrainers,  $N$

Apply Eq. (1.18)

$$N = 870 \text{ kips/in (240 in)} / 14000 \text{ ksi (0.222 in}^2\text{)} = 67.2 \text{ say 68 cables}$$

*References:*

1. California Department of Transportation, Bridge Memo to Designers 20-3 and 20-4
2. UC Berkeley Report No. UCB/EERC 97/12 - "New Design and New Analysis Procedures for Intermediate Hinges in Multiple - Frame Bridges."
3. California Department of Transportation, Seismic Design Criteria