

SUBSURFACE TOMOGRAPHIC IMAGING

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ABSTRACT

Roadway and bridge planning often requires knowledge of the subsurface structure and strength characteristics. The popularity of tomography comes from the ability to obtain images of the subsurface that delineate anomalous areas. In seismic tomography, variations in travel times of elastic waves are most often used as estimators of variation in physical properties, such as density and elastic constants, between the source locations and seismic receivers. Sources and receivers are most often placed in boreholes so that seismic waves pass through the subsurface region between the holes. The travel times of transmitted waves are associated with changes in velocity within the medium along the travel path; i.e., late arrival times mean the seismic energy passed through slow regions and early arrivals relate to fast regions. Variation in seismic velocity is then associated with changes in material properties. In this way, the tomographic data can be used to obtain estimates of engineering properties of the subsurface soil and rock, transforming an image of velocity variation into a geologic map of the subsurface.

INTRODUCTION

Tomographic analyses have been applied in many fields, with tomography a term that has been used to describe a variety of analysis procedures, the most common being medical CAT (Computerized Axial Tomography) scanning. In seismic tomography, the technique normally refers to the measurement of arrival times of traveling compressional waves that pass through the subsurface medium. Variations in travel times are associated with variations in medium velocity or structure. Tomographic experiments that use these compressional wave travel time variations to image structure are known more specifically as acoustic travelttime tomography. Variations on this procedure include 1) using shear waves or surface waves instead of compressional waves, 2) using amplitude data alone (attenuation tomography); 3) using amplitudes as well as travel times of all scattered waves to improve resolution of the intervening medium (waveform tomography); and, 4) to use non-propagating fields (e.g., resistivity tomography). Most often, tomography refers to the measurement of travel time differences for wavepulses transmitted through a medium, and the interpreted velocity variation is associated with physical changes along the ray path as in CAT medical scans. Another variation is to use reflection tomography which estimates medium velocities averaged over the path of reflected seismic events. These act like sound waves and images can be created like sonograms (ultrasounds) of an in-utero fetus or cardiograms of the heart. All tomographic techniques rely on the measurement of variations in some specific parameter, and those variations are associated (mapped) with physical changes within the medium between the source and the receiver. The result is an image of the physical property variation (tomogram).

Ray trace tomography has seen enormous popularity in seismology in recent years, offering a way to invert seismic travel times for an estimate of the background velocities within a propagating medium. Methods often employ straight-rays (Lines and LaFehr, 1989), while others depart from the straight-ray assumption (Bregman, et al., 1989). Travel time information of seismic waves that follow these ray paths, however, can only give smooth variations in the velocities of the medium. To image and invert for discontinuities in the velocity function, researchers use full waveform data, including amplitude information from the seismic signal in the analysis in addition to the travel-time information, and more than just forward-scattered, first arrival travelttime data (Tarantola, 1984; Cardimona, 1991). Diffraction tomography (Devaney, 1984) is a step beyond travelttime tomography toward full waveform inversion, using not just first arrival time data, but phase data from the rest of the scattered wavefield as well. The work of Pratt and Gouly (1991) is an example of a popular scheme, which starts with a tomographic travelttime analysis to get the smooth background velocities, then an imaging step is applied to the full waveform data to get lateral variation in structure.

It is important to be aware of full waveform techniques; however, these techniques suffer from the same limitations inherent in all such geophysical experiments, most importantly the insufficient data coverage of the earth's subsurface. It is imperative that we incorporate into any analysis other geophysical information as constraints to guide the imaging/inversion. What allows us to infer a realistic model for the earth from the inadequate data is our a priori knowledge about physically correct earth models. An important method of specifying the a priori information is through the use of covariance, or smoothing functions (Tarantola, 1984; Cardimona and Garmany, 1993). Other methods involve preselecting appropriate seismic rays before inversion in order to reduce artifacts (Tinti and Ugolini, 1990), or applying separate constraints to each ray in the tomographic analysis (Singh and Singh, 1991).

This paper is meant only as a summary of important tomographic concepts. The reader is referred to Stewart (1991) for a review of general tomography principals. Stork and Clayton (1991) give a review of the methodology of ray trace tomography. Other comprehensive reviews can be found in Kak (1988), Lo and Inderwiesen (1994) and Nolet (1987).

BACKGROUND

Tomographic techniques rely on the measurement of some parameter, and variations in that parameter are associated with, or mapped to physical changes within the medium. In seismic travelttime tomography, we measure travel times of waves that pass through the subsurface medium. Variations in these travel times are associated with variations in elastic wave velocities and are used to create an image of the subsurface. There are two types of body waves that travel through an elastic medium: compressional and shear waves. Compressional waves (P-waves) are most often used for tomographic analysis because P-waves travel faster than S-waves. The P-wave arrives at the receiver first, allowing the interpreter to more easily record an accurate travel time for the wave. Thus, the most common seismic tomographic analysis might more correctly be referred to as seismic P-wave first arrival travelttime tomography, shortened to seismic tomography. Descriptive adjectives are then needed when S-waves, or P-waves other than the first arrival, are used.

Fundamentals

The basic tomographic experiment involves a suite of source-receiver combinations that record signals that have sampled a region of interest (Figure 1). Rays describe how waves propagate between the source and receiver, and the underlying assumption is that the measurements we make are due to variation in the subsurface between the source location and the receiver along the ray between them. The assumption that tomographic measurements are related to variation in the medium along the seismic ray is valid. Although the full seismic waveform may include scattered waves from outside the region between the source and receiver, the travel times of first arrivals should relate to transmitted waves that have passed through the region of interest (e.g, the region between the sources and receivers in Figure 1).

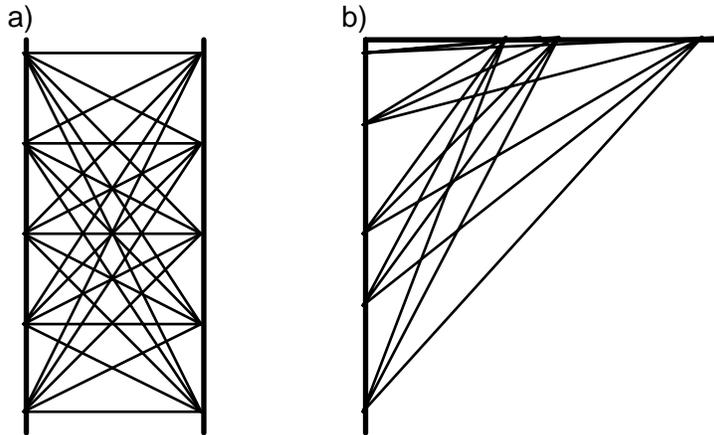


Figure 1. Ray path coverage for a couple simple geometries. a) cross borehole with five source locations and five receiver locations; b) surface to borehole with five borehole locations (source or receiver) and 3 surface locations.

Tomographers often assume that the waves travel along straight lines between source positions and receiver locations (as in Figure 1). The straight line simplification implies the medium is assumed to be homogeneous with only small magnitude perturbations to physical properties between the source and the receiver. This approximation comes out of an underlying principal from Fermat:

** a traveling wave will take the shortest path through a medium.*

The shortest path for the traveling wave would appear to be a straight line between source and receiver. However, Fermat's principal actually deals with the shortest *time* path, which may not be a straight line at all. For seismic tomography, the straight ray assumption is good for creating a first image of structure. The simplification speeds up computations and is usually good for velocity contrasts of less than 10%. The velocity image can later be refined by allowing for curved lines, bending rays, if necessary.

Ray theory, whether with straight or bending rays, is actually a high frequency approximation to full wave theory; i.e., there are no diffractions or other wave phenomena. Wave theory requires knowledge of the whole medium to describe the propagating wave, while ray theory deals only with changes in the medium along the ray. This approximation is reasonable for first arrival traveltime tomography. High frequency waves are assumed, and travel times are affected primarily by the travel path described by the ray. The amplitude of the traveling wave is more sensitive to the region through which the wave propagates, so that the ray theory approximation may be less valid with attenuation tomography or full waveform tomography.

Tomography is an inverse problem, meaning the measurements we make are related directly to specific physical properties of the subsurface. The tomographic measurement (e.g., traveltime) is affected by the physical property variation (e.g., velocity changes) along the length of the ray. When performing the tomographic imaging, this measurement is distributed evenly along the ray back from each receiver to each source. Because of this backprojection, there is an inherent along-path smoothing operation applied in any tomographic experiment. However, the contributions to the tomographic image from each individual source-receiver combination are added together. Resolution is improved, then, via the introduction of more than one direction of illumination. With many angles of coverage, the vertical and lateral shape of anomalies can be more correctly determined.

Example

Figure 2a shows a simple model with a positive (higher than average) velocity perturbation in the center of the region of interest. The model consists of 25 cells with specific velocity perturbation in each. With five sources in a borehole on the left side of the region, and five receivers in a borehole on the right (e.g., Figure 1a), the intervening medium is illuminated by 25 different seismic rays. The travel time of the wave for each ray is an integral of the reciprocal of velocity along the ray length (since distance/rate = time). The travel time will be shorter for those rays that pass through the middle of the region because the wave

travels at a higher velocity part of the time. A representation of the travel time is then backprojected along the ray path between each source and receiver, and contributions from all source-receiver pairs are summed together. The result is an image (Figure 2b) of the velocity variation that created the travel time differences in the data.

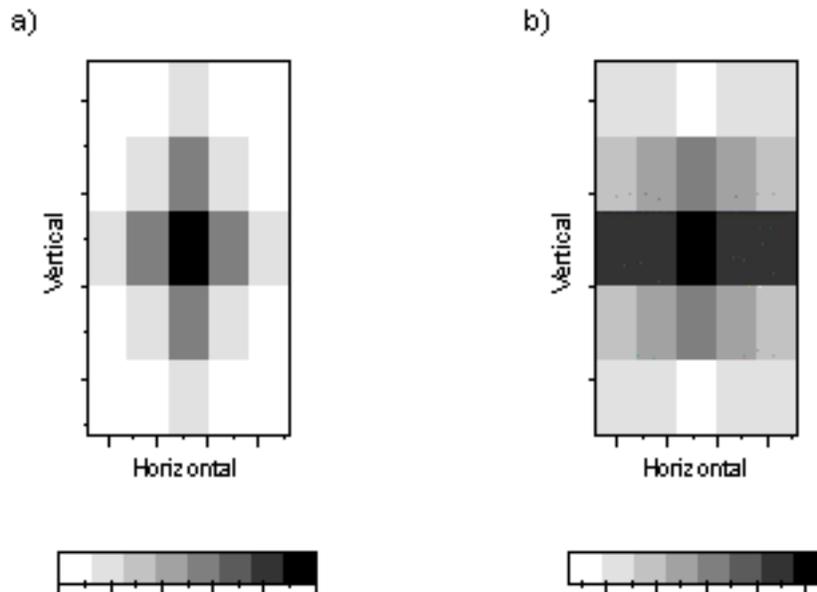


Figure 2. a) Simple 25-cell velocity model with positive velocity anomaly in center; b) tomographic result with five sources in left borehole and five receivers in right borehole for a total of 25 source-receiver pairs.

Clearly, the main high velocity anomaly in the model (Figure 2a) is represented in the image (Figure 2b), however the image is by no means simply a scaled version of the model, which is what we are wanting to achieve. Imperfections in this synthetic case are due to limited angular coverage and uneven ray path coverage (Figure 1a).

Procedure

As shown in Figure 1, the suite of source and receiver locations available will describe the ray-path coverage of the subsurface for the tomographic experiment. The superposition of the interpreted along-ray parameter variation due to each source-receiver combination yields an image of the subsurface. Figure 3a shows the imaging result for the previous example (Figure 2) with data from only one source-receiver combination, and Figure 3b shows the result for one source and five receivers. In general, the more data included, the better representation we have for the subsurface physical property variation. When designing the tomographic survey, we try to obtain as many source and receiver locations as possible, within the constraints of expendable time and effort.

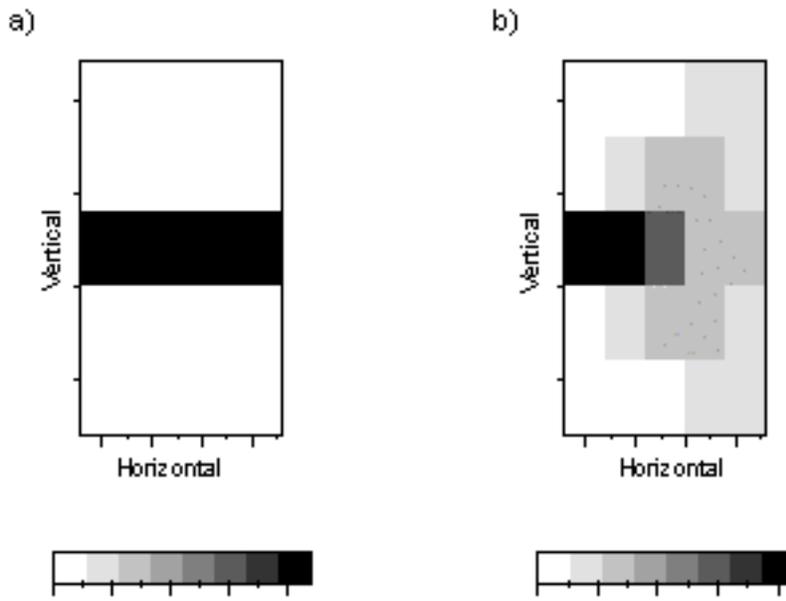


Figure 3. Tomographic results, a) with one source and one receiver at center of right and left boreholes; b) with one source at center of left borehole and five receivers in right borehole.

Specifically, we need at least as many rays (source-receiver combinations) as we have model cells. In Figure 2, the model has 25 boxes, and we illuminate the region with 25 ray paths. Importantly, extra data from similar source-receiver combinations (similar ray paths) do *not* help the tomographic imaging experiment as much as including different angles of coverage. Each ray path should be unique, otherwise the information is redundant. As seen in Figure 2, however, unique ray paths are not sufficient to reproduce the subsurface anomalies in a tomographic experiment. We still may have uneven ray-path coverage of the subsurface, and some angles of illumination that are not utilized at all for the tomographic imaging (Figure 1).

To summarize, the basic tomographic experimental procedure is fairly simple:

- 1) Include many source/receiver combinations to improve resolution;
- 2) Incorporate as many angles of coverage as possible to increase resolution and accuracy;
- 3) Sum variation due to each source/receiver combination;
- 4) Plot variation in cross-section to get tomogram; and,
- 5) Map tomographic image to physical property variation.

INTERPRETATION

Interpretation of tomographic data can be qualitative or quantitative. Often interpretation is limited to simply looking at tomograms to determine anomalous regions. This qualitative interpretation is one of the powerful aspects to tomographic studies. Quantitative interpretation is called inversion. Tomographic data are iteratively compared with synthetic seismograms until they are matched well. The model that produces the seismograms that match the data the best is then chosen as the physical property model to describe the subsurface. As an inverse problem, tomography offers a means to estimate error by noting how closely the synthetic data match the measured data (and/or how closely the chosen subsurface model matches our starting model). This error estimation, which is an important aspect for interpretation, is another advantage of tomographic analysis. Whether qualitative or quantitative, interpretation must take into account a few key points: dominant wavelength, limited angular coverage, uneven ray path coverage, and model discretization.

Dominant wavelength

As with any geophysical technique, there is a fundamental resolving power of the tomographic data. For seismic tomography, this fundamental resolution is related to the Fresnel zone sampling. The Fresnel zone describes how the medium perpendicular to the ray path is averaged by the traveling wave; it is a function of the dominant wavelength and the distance from the source. The farther the target is from the source or the longer the dominant wavelength, the larger the wavefield “footprint” and the worse the resolution. This concept has important implications not only for interpretation, but also for survey design. The dominant wavelength will be a function of the medium velocity and the source frequency, the latter providing us with limited control. Thus, given a specific source and source frequency, we must position our wells appropriately based on the wavefield footprint and our intended resolution. Alternatively, given a specific positioning of wells, we must choose our source appropriately to satisfy our expected resolution.

Limited angular coverage

Limited angular coverage by ray paths reduces the lateral resolution in the tomogram. Because of the inherent along-ray path smoothing, having different angles of coverage improves the 2 and 3 dimensional resolution of any subsurface anomalies (Figure 2 and 3). The caveat here is that limited angular coverage will cause artifacts in the tomographic images. This defines one of the main interpretational challenges after any tomographic survey. Comparing the result from one ray path (Figure 3a), five ray paths (Figure 3b), and 25 ray paths (Figure 2b), there are clear differences in the images created in each case. None represent the true subsurface anomaly correctly (Figure 2a). These differences are due in large part to limited angular coverage. The limitations of the source-receiver geometry must be understood before any interpretation can be made.

With sources and receivers in boreholes (i.e., none at the surface), it is difficult if not impossible for tomography to resolve lateral (horizontal) variations. There will always be a horizontal component of smoothing that does not improve through the process of superposing data from all source-receiver combinations. This will be true even when the model is overdetermined (i.e., there are at least as many ray paths as there are model cells).

Uneven ray path coverage

Even with a nominally well-determined problem (25 rays of Figure 2b for the 25 model cells), there are artifacts in the image due to low ray coverage of parts of the model. In other words, both the number of rays and the angles of the ray paths are important for creating a well-determined tomographic experiment. Note in Figure 1 that the highest concentration of ray paths is vertically along the center of the region between the two boreholes. This results in the better imaging in Figure 2b of this same central region. Tomographic images will be biased toward areas that have more ray coverage. Interpretation of any tomographic image or inversion must include a discussion of ray path coverage in order to know which parts of the resulting model (based on the image or inverted for directly) are best constrained by the data. This will be true for straight or bending ray tomography.

Model discretization

Data resolution will be limited by the model parameter discretization. If the subsurface is made up of a blocky model, as in our synthetic example in this paper (Figure 2a), then a blocky image will be a good representation (Figure 2b). However, geological variation is often more gradual. The blocky nature to the imaging comes from our model discretization. After an imaging experiment, we can smooth the image (Figure 4a), and hence artificially “improve” resolution of the interpreted velocity model (Figure 4b). In an inversion, we can explicitly increase resolution by choosing smaller model discretization before performing forward modeling calculations to compare with the data. Either way, the model discretization must be chosen based on the limitations of the data. The data must be able to represent the model parameters well (over-determined problem), and the subsurface anomalies of interest must be resolvable.

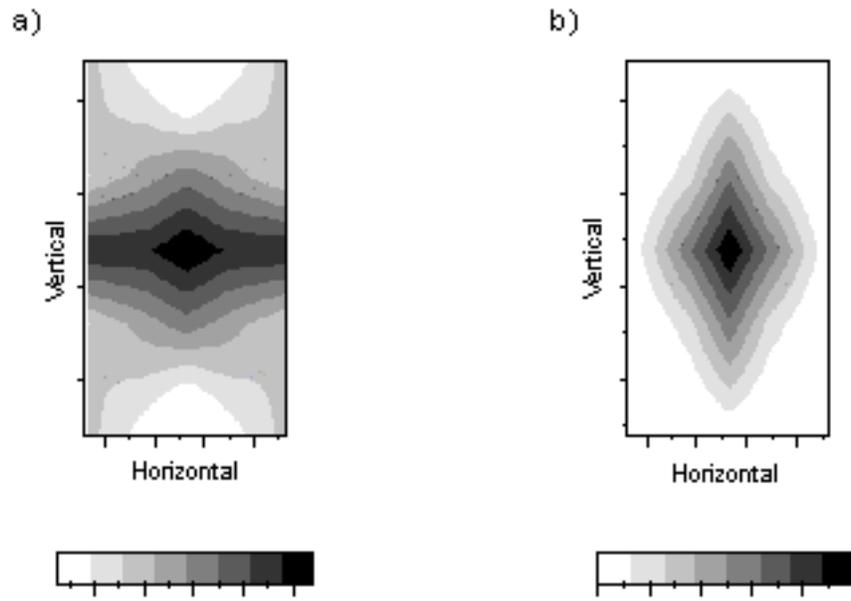


Figure 4. a) Tomographic result of Figure 2b) with image smoothing applied; b) smoothed velocity model interpreted from smoothed image (to compare with Figure 2a).

CONCLUSION

Seismic travelttime tomography is a method used to produce a two or three dimensional image of a region of the subsurface. Variations in travel times are used as estimators of variation in physical properties between the seismic source and receiver locations. Seismic travelttime tomographic techniques suffer from limitations inherent in all such geophysical experiments, most importantly the insufficient sampling of the earth's subsurface. It is important that we incorporate into the analysis any other geophysical or geotechnical information as constraints to guide the imaging or inversion. What allows us to infer a realistic model for the earth from the inadequate data is our a priori knowledge about physically correct earth models. The power of seismic tomography, then, is the ability to obtain images of the subsurface that delineate anomalous areas. The travel times of transmitted waves are associated with changes in velocity within the medium along the travel path, and variation in seismic velocities can then be associated with changes in material properties (density and elastic constants). The tomographic data are used to obtain estimates of physical properties of the subsurface soil/rock, turning a tomographic image into a geologic map of the subsurface.

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